clusters, assistance towards fixed costs and profit making, costs, profitability of systems

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RESEARCH THE BEHAVIOR OF THE ASSISTANCE TOWARD FIXED COSTS AND PROFIT MAKING

Abstract

Contemporary science and business regard controlling as the inseparable part of the modern management concept.

The current scientific and practical knowledge of industrial engineering was used as a basis for identification the main tasks of controlling management in an enterprise. The analysis of controlling tasks frequency shows, that the cost calculation and the performance calculation belong among the most frequent tasks, and that there is some kind of evaluation for all (production or non-production) areas in enterprises.

Lean production is characterized by elimination of redundant, useless or partly necessary processes. Lean production is considered as the basis for the network enterprises and clusters. Individual enterprises are selected to clusters on the basis of rules of outsourcing.

1. INTRODUCTION

The 21st century companies are the part of the worldvide complex, which changing conditions the companies have to conform in the shortest possible time. The constantly increasing complexity and dynamics of the market environment poses (presents) the company to the merciless pressure of competitors. Every day the managers find them in such situations, when they need to decide promptly and in the right way. The successfulness of the companies is the matter of the permanent improving of the management system on the basis of implementation and using the modern approaches and methods of industrial engineering and management.

One possible reason for the loss of the competitiveness and the consequential crisis of the company are the bad decisions in cost management. Effective management could be reached by application of operative controlling methods, which can identify critical places in companies in time. Controlling methods are not only part of the internal audit, but they also play a big part in managing and planning of production units and they set strategic and operative targets.

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2. PROGNOSTIC MODEL CREATING

The current scientific and practical knowledge of industrial engineering was used as a basis for identification the main tasks of controlling management in an enterprise. The analysis of controlling tasks frequency shows, that the cost calculation and the performance calculation belong among the most frequent tasks. The elements of econometric analysis were used to reach the target – research the behavior of the assistance toward fixed costs and profit making in relation to changing input conditions in vertical and horizontal level in production units (workshops, enterprises, clusters, etc.).

Prognostic model will be created and will be used for identification of current situation, estimating the future situation and searching for the best solution with changing input parameters.

2.1. Setting the main methods for reaching the target

Production units can be analyzed from various aspects. If costs are considered as the evaluation criterion for subjects' economy, the cost analysis is essential for efficient managing of the assistance toward fixed costs and profit making. Reached costs value of production unit depends on chosen managing method, technology, production organization, production factors consumption, and on the conditions of the whole process. It is necessary to formulate relations and interdependence among the elements in the analytic kind of work, based on the relation „cause – effect". Relations among the elements can be described by econometric methods and models or by economic-statistic methods. Econometric methods use mathematical functions to describe the economic relations.

2.2. Cost function analysis

Cost functions describe the relation between costs "N" and production volume "q". General mathematical formula of cost function:

$$CN = aq + b \tag{1}$$

q - Independent variable
CN - Total costs, dependent variable
Parameters a, b - constants

Mathematical formula of cost function:

$$CN = f(q) \tag{2}$$

There are several types of cost behavior:

1) Linear costs – rise proportionately to production volume;

2) Progressive costs – rise faster than production volume;

3) Degressive costs – production volume rise faster than costs;

4) Progressive – degressive costs – inflection point is the point, in which the cost behavior is changing;

5) Fixed costs – constant to the production volume;

6) Discontinuous costs;

7) Regressive costs – decreasing with increasing production volume, rare.

Various combinations of cost behavior are the subject of economic analysis, target of the analysis is to show, whether the current production situation in chosen production process is optimal as for production volume. Capacity of chosen production unit is given and quantity q can be chosen from the range of given production unit capacity.

Tabular summary of cost functions

Cost behavior can be described by mathematical analysis, by cost functions.

Tab.1. Total costs function

Total costs function	Cost behavior
$FN = b$	Fixed costs
$VN = aq$	Linear variable costs
$VN = aq^2 + bq$	Progressive variable costs
$VN = -aq^2 + bq$	Degressive variable costs
$CN = aq + b$	Liner total costs
$CN = aq^2 + bq + c$	Total progressive costs
$CN = -aq^2 + bq + c$	Total degressive costs
$CN = aq^3 - bq^2 + cq + d$	Total progressive-degressive costs
$CN = a - bq$	Total regressive costs

2.3. Yield function analysis

Yield function is opposite to cost function. Also yield function depends on production volume. Mathematical formula:

$$CV = f(q) \tag{3}$$

CV - Total yield
q – Production volume

Yield behavior can be theoretically similar to cost behavior, only without fixed part. Yield can be:

1) Linear: $CV = aq$ (4)

2) Progressive: $CV = aq^2 + bq$ (5)

3) Degressive: $CV = -aq^2 + bq$ (6)

4) Progressive–degressive $CV = aq^3 + bq^2 + cq$ (7)

5) Increasing and decreasing: $CV = -aq^2 + bq$ (8)

In economic analysis yield depends on unit price and volume. Price behavior depends on demand function, which describes price dependence on volume:

$$p = f(q) \tag{9}$$

Demand function can be generally characterized as decreasing (price must decrease with increasing volume if demand should be kept). Yield is characterized as the volume function, it is possible to describe yield by following function:

$$CV = f[\varphi(q)] \tag{10}$$

Demand functions are not only price functions but also average yield PV, because of formula:

$$CV = PV * q \tag{11}$$

$$PV = CV / q \tag{12}$$

$$CV = p * q \tag{13}$$

$$p = PV = f(q) \tag{14}$$

Demand function is a basis for cost analysis, because total costs can be set as the multiple of demand function and volume unit q.

To explain yield behavior it is necessary to consider also average and differential costs. If the price is constant, then:

$$CV = p * q$$

p – price - constant.; (variant 1).

For progressive yield the price increase in needed 2), but it is not very common in practice, because price decreases with increasing demand. The same situation is in variant 4), in which the price increase is expected:

2) $CV = q(aq + b) = aq^2 + bq$

4) $CV = q(aq^2 + bq + c) = aq^3 + bq^2 + c$

The most common for yield behavior is the last variant:

5) $CV = q(-a + bq) = -aq^2 + bq$

Variant 5) is often used in forming the optimal yield situation in company.

2.4. Mathematical formulation of assistance toward fixed costs and profit making model

Phases of assistance toward fixed costs and profit making behavior:

1) General model creating
2) General, multistage model creating

Horizontal level is used for studying assistance toward fixed costs and profit making in one production unit, which can produce one or more products. Vertical level is used in the second model, which is proposed for studying assistance toward fixed costs and profit making behavior in multistage production units, which can produce one or more products.

2.5. General model

General model is proposed for evaluating of one final product in studied production unit. In studied economic functions their extreme values are important. Assistance toward fixed costs and profit making model extreme is set by differential calculus application.

Every econometric model must have two types of equations - behavior equation and definition equation. Number of equations depends on the number of variables. The endogen and exogenous variables must be defined. Number of variables must be the same as number of equations.

Anatomy of assistance toward fixed costs and profit making model is described by system of equations:

1. $VN = f(q)$	behavior equation	(16)
2. $p = f(q)$	behavior equation	(17)
3. $CV = p * q$	definitional equation	(18)
4. $PÚ = CV - VN$	definitional equation	(19)
5. $PÚ_{\max} = \max(PÚ)$	definitional equation and balance condition	(20)

Model comprises 5 variables:

- Four endogen $CV, CN, PÚ, q$
- One exogenous: p

Production capacity - variable *q,* is the maximal production volume, which can be produced by production unit in certain time (year, day, hour, etc.). Generally can be production capacity of production unit characterized as the result of its performance and time of activity.

Function analysis – assistance toward fixed costs and profit making behavior

Analysis of cost and yield behavior can describe optimal situation in company. Optimal situation in company is set by the proposed general model of assistance toward fixed costs and profit making, in which cost and yield behavior is specified by cost and yield functions.

Linear costs and yield

Critical point is set when following condition is fulfilled:

$$CV = VN \quad (21)$$

Equation can be controlled by economic rule:

$$PÚ_{\max}, q_{opt} \rightarrow DV = PV = DVN \quad (22)$$

DV – differential yield

DVN – differential variable costs

PV – average yield

Critical point in this variant occurs, when production volume is zero, because assistance toward fixed costs and profit making does not contain fixed costs. If assistance toward fixed costs and profit making is minus, variable costs are not fully covered. In this case it is not recommended to produce the product, if it is not important from the strategic point of view. If assistance toward fixed costs and profit making value is plus, the total value of assistance toward fixed costs and profit making will increase with increasing production volume to the maximal production capacity.

Assistance toward fixed costs and profit making value should cover the fixed costs and profit, because:

$$PÚ = FN + Z \tag{23}$$

Z – Profit

FN – Fixed costs

If the production capacity of production unit is not fully employed, other orders can be accepted, even though they are not profit making or do not fully cover fixed costs. It is important to employ the production capacity as much as possible and to split fixed costs into many products.

Graphic model figure

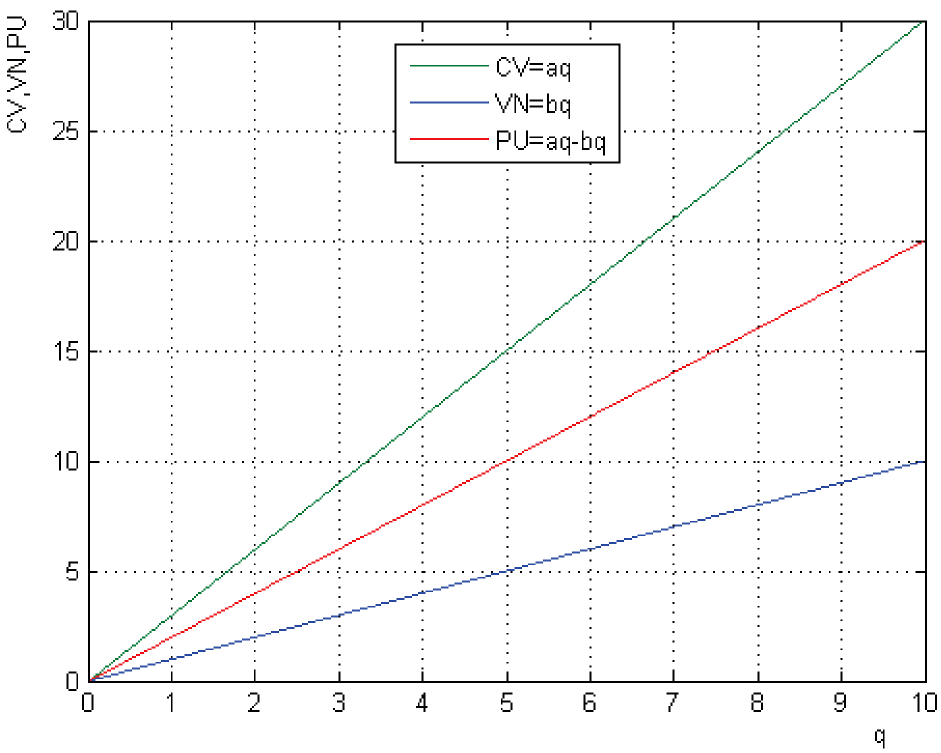

Fig.1. Total functions at chosen parameters of variables a, b

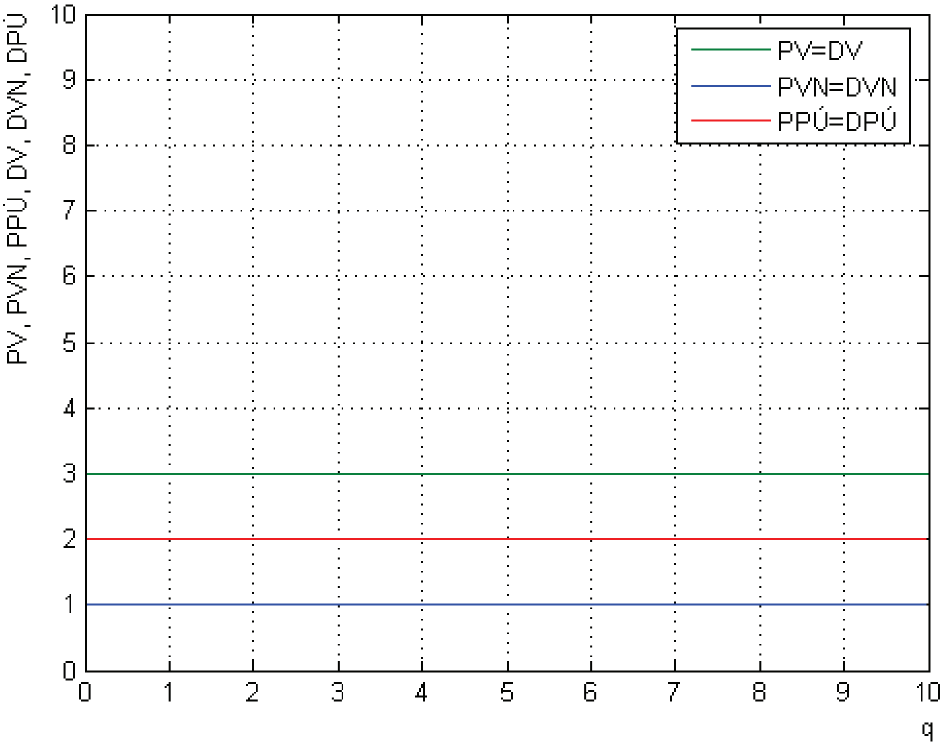

Fig.2. Differential and average functions at chosen parameters of variables a, b

Non-linear costs and linear yield

This variant is often used in company practice.

- Price per unit is constant, variable costs are degressive.
- Price per unit is constant, variable costs are progressive.
- Price per unit is constant, variable costs are degressive – progressive with inflection point .

Step 1: finding analytical form of cost and yield function and of average variable costs (PVN) and differential variable costs (DVN).

Step 2: setting total and average assistance toward fixed costs and profit making at given price.

Step 3: to prove, that DVN line is maximal or minimal line of assistance toward fixed costs and profit making.

Step 4: finding such a production volume q, when maximal value of assistance toward fixed costs and profit making is reached. Assistance toward fixed costs and profit making value is constantly increased till value of differential income is higher then value of differential costs.

Optimal situation can be set:

a) By calculation of total assistance toward fixed costs and profit making. Maximal assistance toward fixed costs and profit making is reached If differential assistance toward fixed costs and profit making is zero:

$$P\acute{U}' = \frac{dP\acute{U}}{dq} = 0 \Rightarrow q_{opt} \Rightarrow P\acute{U}_{\max} \quad (24)$$

b) By comparing differential costs with relevant price. At constant prices production unit reaches maximal value of assistance toward fixed costs and profit making in the point:

$$p = PV = DVN \Rightarrow q_{opt} \quad (25)$$

Graphic model figure

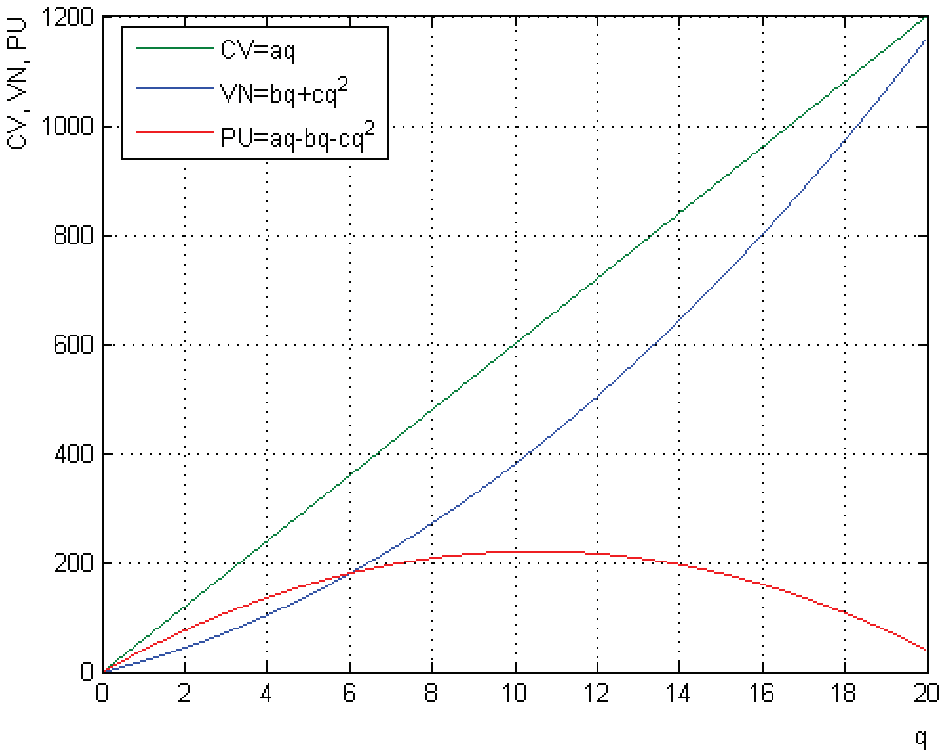

Fig.3. Total functions at chosen parameters of variables a, b, c

It is clear from figure 4, that for setting the maximal value of assistance toward fixed costs and profit making the price behavior is important. Intersection point of the price and of the differential variable costs sets the optimal production volume q. Assistance toward fixed costs and profit making behavior is not important for finding the optimal situation.

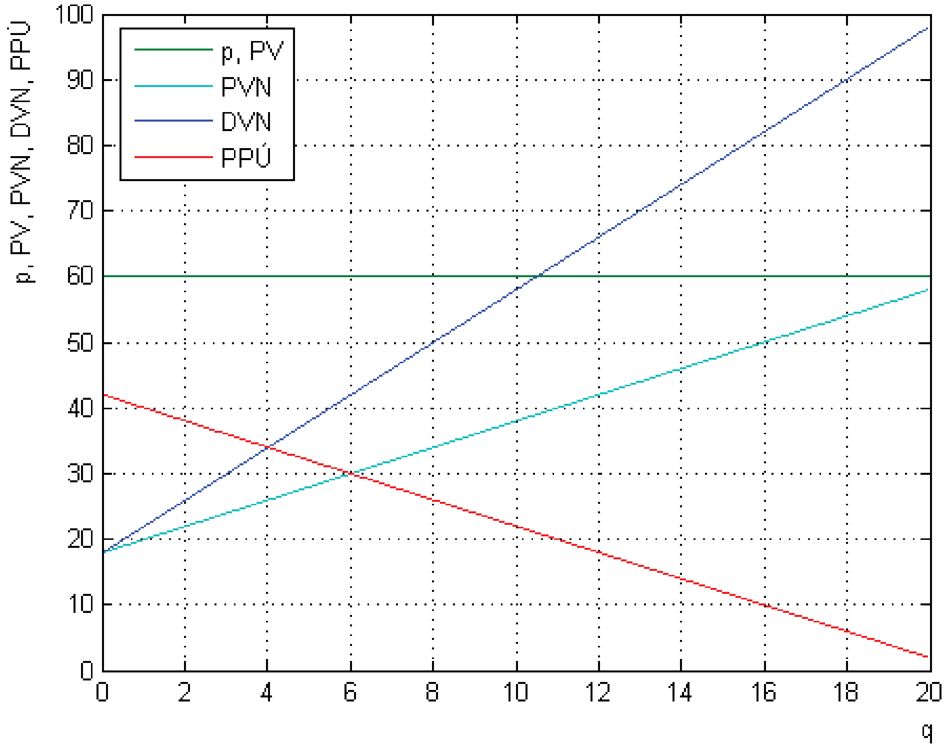

Fig.4. Differential and average functions at chosen parameters of variables a, b, c

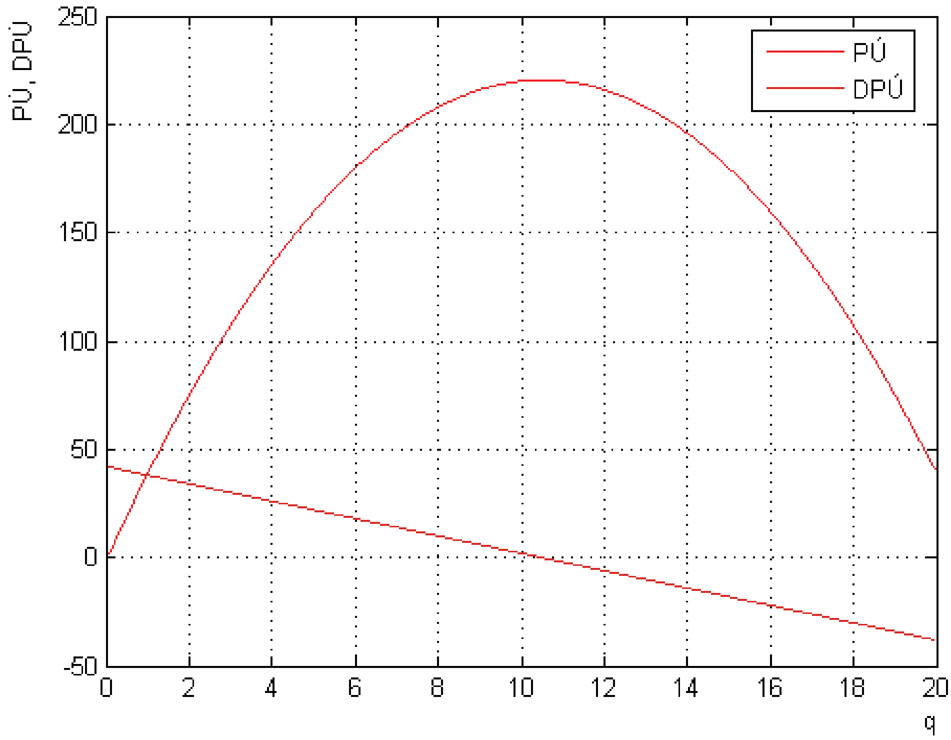

Fig.5. Total and differential function of assistance toward fixed costs and profit making at certain parameters of variables a, b, c

Optimal production situation (maximal value of assistance toward fixed costs and profit making) is reached in the intersection point of DPU and line q.

Linear costs and non-linear yield

Setting the maximal value of assistance toward fixed costs and profit making is the same as in variant non-linear costs and linear yield.

Graphic model figure

Absolute value of optimal assistance toward fixed costs and profit making is clear from the total functions figures and differential function figures. In the total functions figure 6 can be maximal assistance toward fixed costs and profit making set as the difference between total costs and total variable costs in the point of optimal volume, or as the difference between total assistance toward fixed costs and profit making and line *q* in the point of optimal volume.

Average assistance toward fixed costs and profit making can be also set from the figure 7 as the difference between average costs and average yield in the point of optimal volume. Optimal volume is in the intersection point of DV and DVN. Assistance toward fixed costs and profit making behavior is not important for finding the optimal situation.

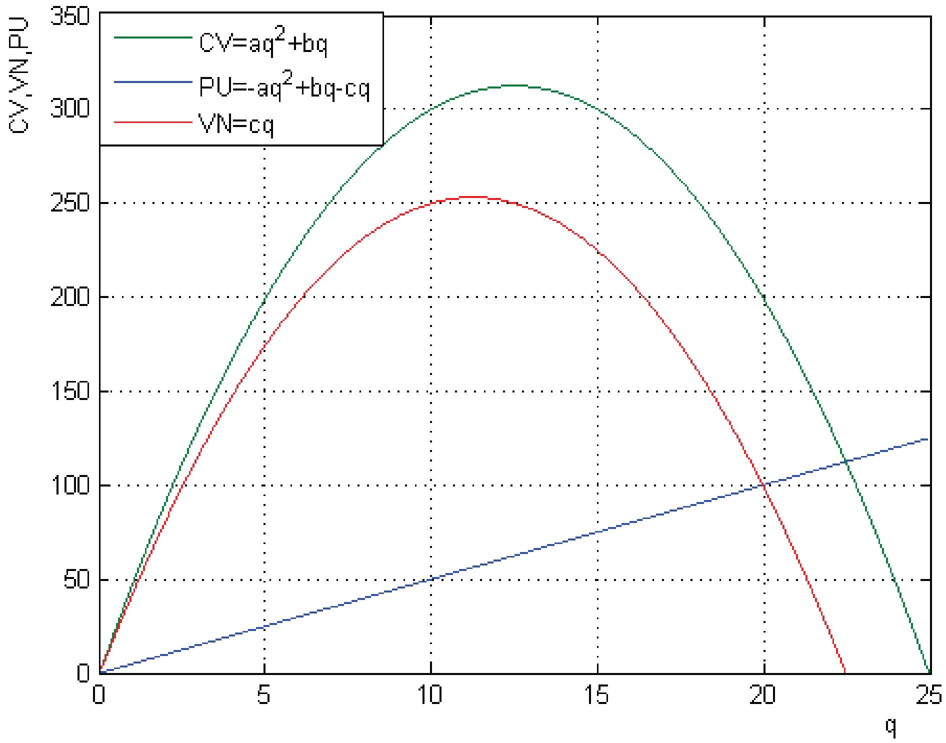

Fig.6. Total functions at chosen parameters of variables a, b, c

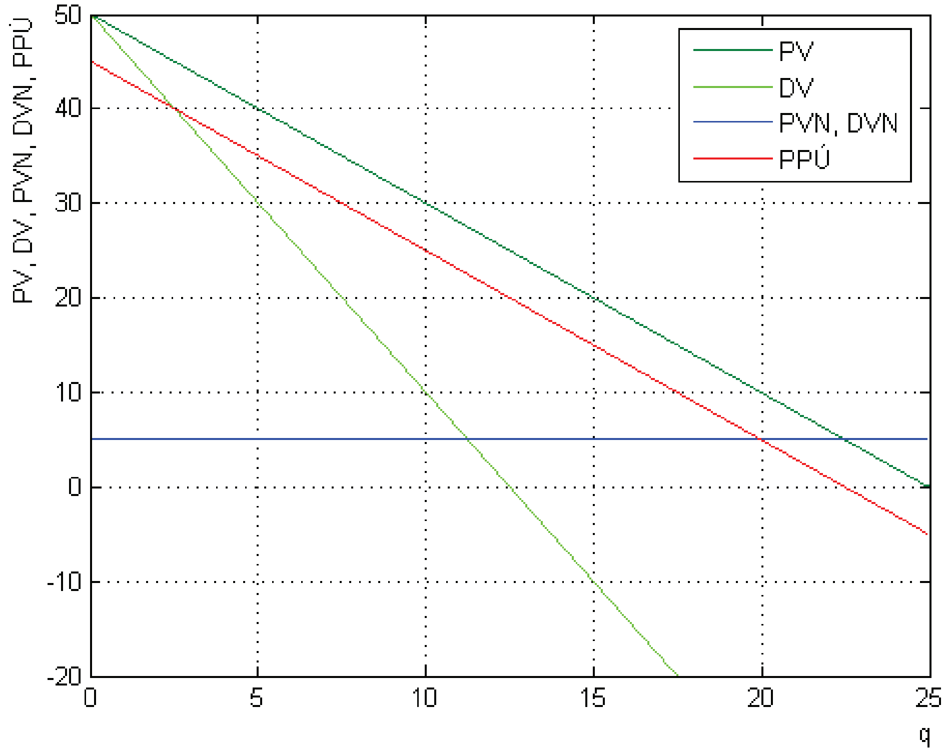

Fig.7. Differential and average functions at chosen parameters of variables a, b, c

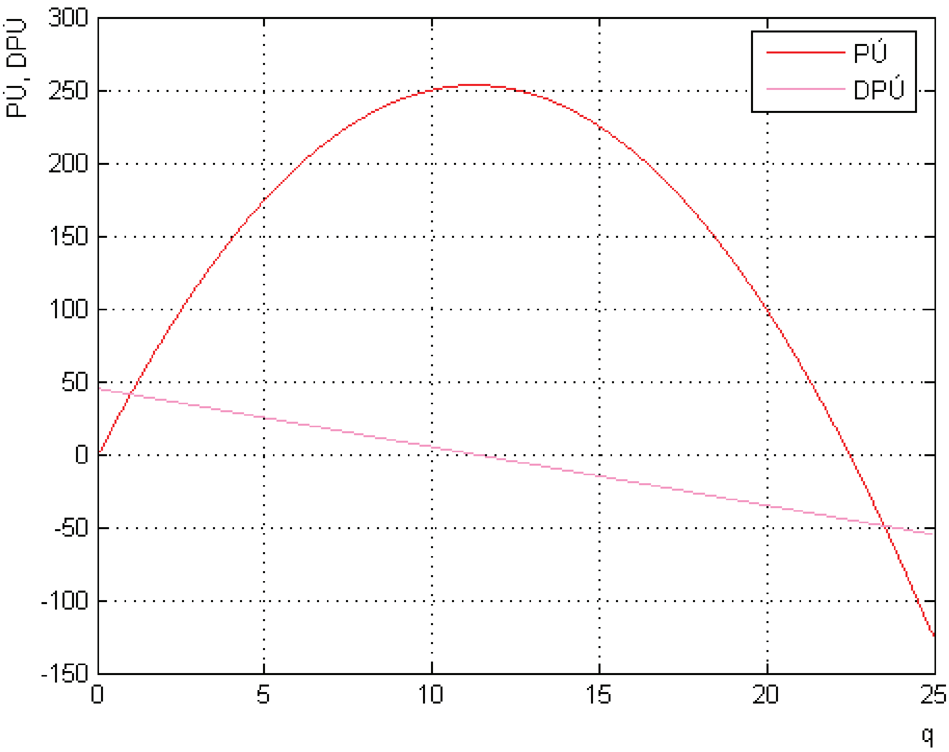

Fig.8. Total and differential function of assistance toward fixed costs and profit making at certain parameters of variables a, b, c

More efficient is behavior of differential assistance toward fixed costs and profit making. It is clear from the figure 8, that relation between assistance toward fixed costs and profit making behavior and differential assistance toward fixed costs and profit making behavior is important. Optimal production situation (maximal value of assistance toward fixed costs and profit making) is reached in the intersection point of *DPU* and *q*.

Non-linear costs and non-linear yield

Non-linear costs and yield variant is the most complicated situation, when price depends on demand function. Setting maximal value of assistance toward fixed costs and profit making is the same as in the variant non-linear costs and linear yield.
The most general variant allowing set not only total optimum and critical points, but also all partial optimums.

Average minimal costs

$$PN_{\min} \Leftrightarrow DVN = PVN \tag{25}$$

Total maximal yield

$$CV_{\max} \Leftrightarrow DV = 0 \tag{26}$$

Average maximal assistance toward fixed costs and profit making

$$PPÚ_{\max} \Leftrightarrow DPÚ = PPÚ \tag{27}$$

Graphic model figure

Absolute volume of optimal assistance toward fixed costs and profit making is clear from the total functions figure and from the differential functions figure. Setting the maximal assistance toward fixed costs and profit making from the total functions figure 9 is the same as in variant 3.

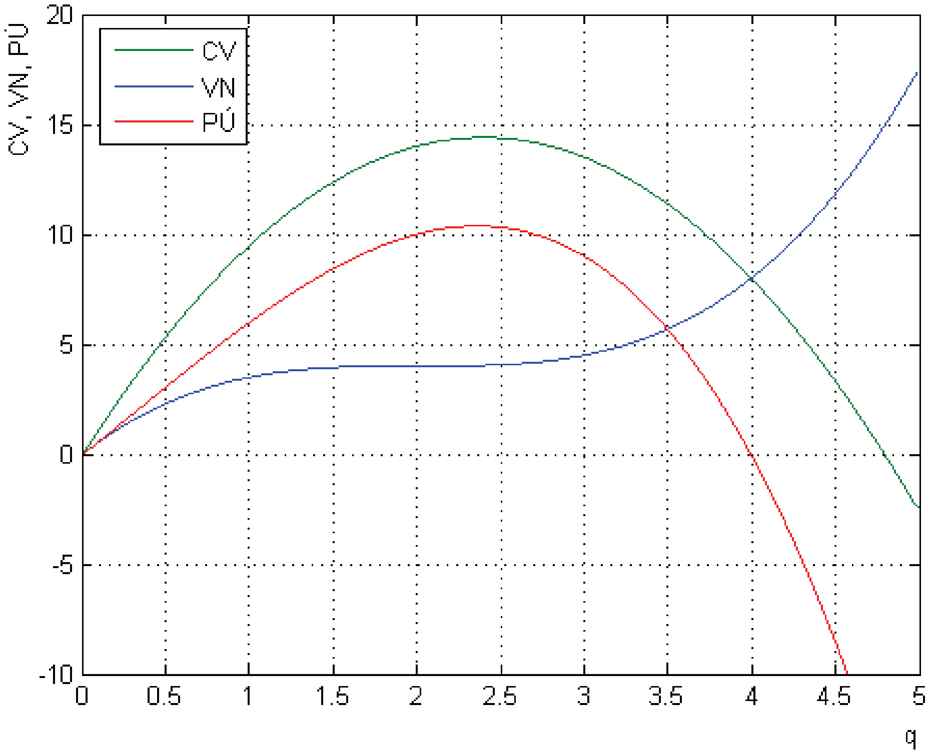

Fig.9. Total functions at chosen parameters of variables a, b, c, d, e

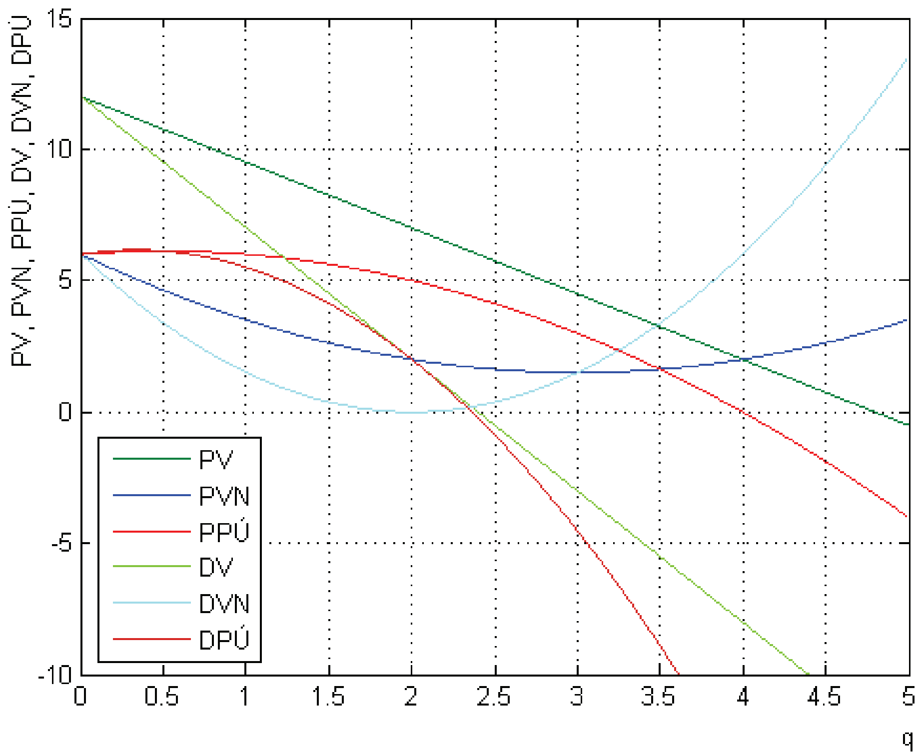

Fig.10. Differential and average functions at chosen parameters of variables a, b, c, d, e

In figure 10, it is possible to set average assistance toward fixed costs and profit making as the difference of average costs and average yield in the point of optimal volume. Optimal volume is in the intersection point of DV a DVN. Assistance toward fixed costs and profit making behavior is not important for finding the optimal situation.

The partial optimums can be also found in the figure – minimal average costs, maximal total yield and maximal assistance toward fixed costs and profit making at the certain production volume q.

Relation between assistance toward fixed costs and profit making behavior and differential assistance toward fixed costs and profit making is important again.

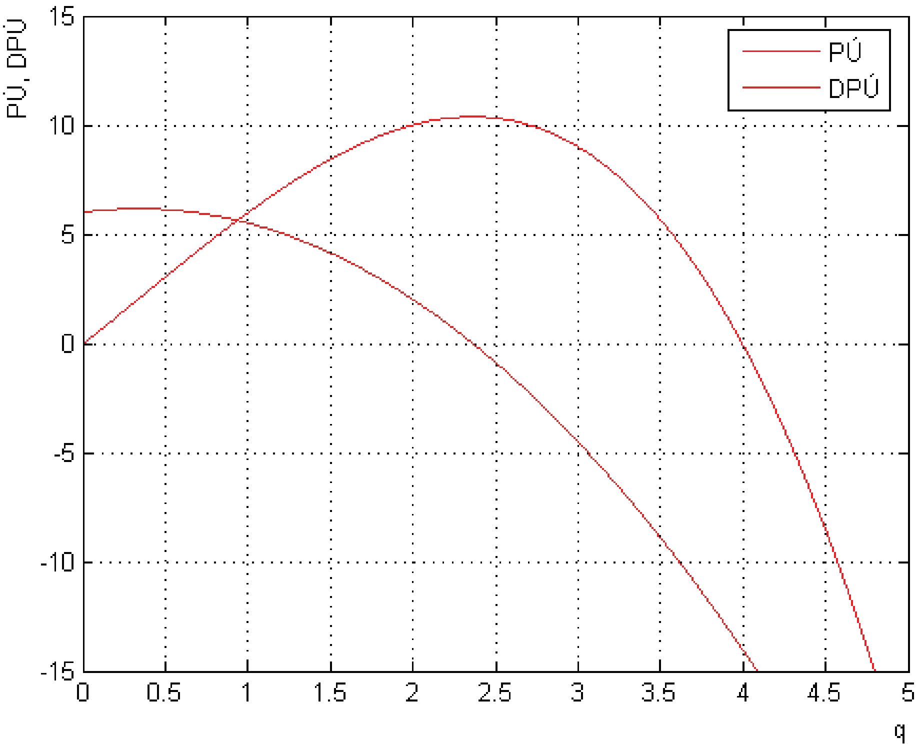

Fig.11. Total and differential function of assistance toward fixed costs and profit making at certain parameters of variables a, b, c, d, e

Yield and cost functions can be in the whole production capacity defined also by more functions with more partial intervals.

Limiting conditions

Heaviside function is used for mathematical formulation of cost and yield functions in more partial definitional fields. General formulation of a partial function by Heaviside function:

$$f_j(q_i)*h(q_i - q_{i,j-1}) - f_j(q_i)*h(q_i - q_{i,j}) \quad (28)$$

General formulation of all partial functions in the whole definitional field by Heaviside function:

$$f^i(q_i) = \sum_j f_j(q_i) * h(q_i - q_{i,j-1}) - f_j(q_i) * h(q_i - q_{i,j}) \qquad (29)$$

$h(q)$ - Heaviside function for mathematical formulation of partial cost functions and their types

$g(q)$ - Heaviside function for mathematical formulation of partial yield functions

2.6 General, multistage model creating

Multistage model is meant for multistage systems of production units (division, company, cluster, etc.), which can produce one or more products. Model is studied both in vertical and horizontal level.

Model is proposed for two stages of managing. The first, initial stage is production unit, the second system of production units (e.g. cluster). Similarly could be defined e.g. three-stage model for workshop, company and cluster.

Limiting conditions, which must be kept:

- Choosing the basic production unit for setting the assistance toward fixed costs and profit making in horizontal level.
- Presumption of flat cost accounting.
- Presumption of flat standardization.
- Presumption of reciprocal account (avoiding the duplicity of account on the higher level – condition of profit centers).

Demand function of product is defined by more demand functions in more definitional fields. Total variable cost function is defined as the sum of partial types of variable costs. Every type can be defined by more cost functions in more definition fields. Total variable costs CVN and total yield CV on the highest chosen level are set by summary function.

Proposed model of assistance toward fixed costs and profit making is described by the system of equations for two stages of managing:

${}^{l}_{k}vn^{i}_{j}\left({}^{l}q^{i}_{j}\right)$ Formulation of *l*th partial type of variable costs of *k*th type of variable costs in *j*th product *i*th production unit.

$$CVN = \sum_{i=1}^{n}\sum_{j=1}^{m_i}\sum_{k=1}^{r^i_j}\sum_{l=1}^{{}_k p^i_j}\begin{bmatrix} {}^{l}_{k}vn^{i}_{j}\left(q^{i}_{j}\right) * h(q^{i}_{j} - {}^{l-1}q^{i}_{j}) - \\ -{}^{l}_{k}vn^{i}_{j}\left(q^{i}_{j}\right) * h(q^{i}_{j} - {}^{l}q^{i}_{j}) \end{bmatrix} \quad \text{behavior equation} \qquad (30)$$

$$p = f(q)$$ behavior equation (31)

${}^{s}cv_{j}^{i}\left({}^{s}q_{j}^{i}\right)$ Formulation of *s*th partial total yield of *j*th *product in i*th production unit.

$$CV = \sum_{i=1}^{n}\sum_{j=1}^{m_i}\sum_{s=1}^{t_j^i}\begin{bmatrix} {}^{s}cv_{j}^{i}\left(q_{j}^{i}\right) * g(q_{j}^{i} - {}^{s-1}q_{j}^{i}) - \\ - {}^{s}cv_{j}^{i}\left(q_{j}^{i}\right) * g(q_{j}^{i} - {}^{s}q_{j}^{i}) \end{bmatrix}$$ definitional equation (32)

$$P\acute{U} = CV - CVN$$ definitional equation (33)

$$P\acute{U} = \sum_{i=1}^{n}\sum_{j=1}^{m_i}\sum_{s=1}^{t_j^i}\begin{bmatrix} {}^{s}cv_{j}^{i}\left(q_{j}^{i}\right) * g(q_{j}^{i} - {}^{s-1}q_{j}^{i}) - \\ - {}^{s}cv_{j}^{i}\left(q_{j}^{i}\right) * g(q_{j}^{i} - {}^{s}q_{j}^{i}) \end{bmatrix} -$$

$$- \sum_{i=1}^{n}\sum_{j=1}^{m_i}\sum_{k=1}^{r_j^i}\sum_{l=1}^{{}_{k}p_j^i}\begin{bmatrix} {}_{k}^{l}vn_{j}^{i}\left(q_{j}^{i}\right) * h(q_{j}^{i} - {}^{l-1}q_{j}^{i}) - \\ - {}_{k}^{l}vn_{j}^{i}\left(q_{j}^{i}\right) * h(q_{j}^{i} - {}^{l}q_{j}^{i}) \end{bmatrix}$$

$$P\acute{U}_{\max} = \max\left(P\acute{U}\right)$$ definitional equation and balance condition (34)

3. CONCLUSION

The basis for development of company evaluation in cost management is econometric analysis, which allows gaining complex information about economic situation in company from isolated economic data. Knowledge of relations and interdependency in the company processes is necessary for controlling.

Proposed prognostic model can be used for identification of current situation and also for estimating the future trend at changing input parameters. Described models differ in the number of evaluated products and in number of managing stages. The input values for models are the cost and yield functions. The key criterion for decision about the production programme is in both models value of assistance toward fixed costs and profit making and its maximization.

4. ACKNOWLEDGEMENT

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